South Orange & Maplewood School District
Mathematics Curriculum Grade 11-12
Topics in Calculus

South Orange Maplewood School District
Department of Curriculum & Instruction
December 2013

TABLE OF CONTENTS

ACKNOWLEDGEMENTS
Curriculum Writers
Candice Beattys
Megan Power
Elaine Weiland

Course Title: Topics in Calculus (Level 3 or Level 4)

Length of Course: One semester (2.5 credits)

Prerequisites: Precalculus and Functions Analysis

Description:
This course provides a preliminary introduction to Calculus for students who will likely specialize in business, economics, management, life and social sciences. Calculus plays an important role in these areas. It is the mathematics of change, and we live in a constantly changing world.

The goal of this course is to equip students with the powerful tools of Calculus. At the foundation of mathematics is the idea of a function. Functions express the way one variable quantity is related to another quantity. Calculus was invented to deal with the rate at which a quantity varies, particularly if that rate does not stay constant. This course begins with a thorough review of functions, particularly the properties, behavior and manipulation of the polynomial function. However, the review will be approached through the lens of calculus, which will provide a unifying perspective for all functions. Beyond this enhanced review, students will construct a firm understanding of the derivative and its applications.

This course is designed to give students a proper balance between the mastery of skills and the acquisition of key concepts. This curriculum will be guided by two principles. The first is the *Rule of Three* which requires that every topic be presented geometrically, numerically and algebraically. The second guiding principle is the *Way of Archimedes* which states that formal definitions and procedures evolve from the investigation of practical problems. Specifically, this curriculum provides many problems that are applications of the Social Sciences, Life Sciences and Business arenas and are generally accepted as important. This course exceeds requirements specified in the CCSS and State Standards.

Evaluation:
Student performance will be measured using a variety of instructor-specific quizzes and chapter tests as well as departmental common assessments and Final Exam. Assessments will balance the degree to which required concepts and skills have been mastered.

Text:
Calculus: Concepts & Contexts 3rd Edition, James Stewart, Thomson Brooks/Cole 2005

Reference Texts:
Calculus, Deborah Hughes-Hallett, Andrew M. Gleason, et al, Wiley & Sons, 1994
Calculus: Graphical, Numerical, Algebraic, Ross Finney, Franklin Demana, et al, Pearson/Prentice Hall, 2007
Applied Calculus, Bernard Kolman & Charles G. Denlinger, 1989
Humongous Book of Calculus Problems, W. Michael Kelley, 2007

Learning Objective The student will…	Content Outline	Instructional Materials	Standards for Mathematical Practice
1. Evaluate and interpret the limits of a function.	1. Investigate the connection between how a function is defined with regard to the existence of a limit as x approaches some designated value. 2. Connect and identify the limit of a function given its graph 3. Connect and identify the limit of a function given its equation 4. Determine how and when a limit can fail to exist. 5. Recognize and identify values at which the limit of a function does not exist 6. Identify limits in a piecewise function 7. Sketch a function given information about its limits 8. Use a graphing calculator to evaluate limits of a function 9. Find limits using direct substitution and factoring	*Calculus: Concepts & Contexts* Sections 2.2- 2.3 District constructed materials	1. Make sense of problems and persevere in solving them. 2. Reason abstractly and quantitatively. 3. Construct viable arguments and critique the reasoning of others. 4. Model with math. 5. Use appropriate tools strategically. 6. Attend to precision. 7. Look for and make use of structure. 8. Look for and express regularity in repeated reasoning.

Learning Objective The student will…	Content Outline	Instructional Materials	Standards for Mathematical Practice
2. Represent and evaluate problems involving continuity.	1. Investigate and identify where a function is continuous given its graph or its algebraic equation 2. Explain how and why the concept of limits and continuity are related. 3. Sketch functions given information about intervals of continuity 4. Investigate and identify values for constants so that a piecewise function will be continuous at a given point 5. Apply the Intermediate Value theorem to functions and their graphs	*Calculus: Concepts & Contexts* Section 2.4 District constructed materials	1. Make sense of problems and persevere in solving them. 2. Reason abstractly and quantitatively. 3. Construct viable arguments and critique the reasoning of others. 4. Model with math. 5. Use appropriate tools strategically. 6. Attend to precision. 7. Look for and make use of structure. 8. Look for and express regularity in repeated reasoning.

Learning Objective The student will…	Content Outline	Instructional Materials	Standards for Mathematical Practice
3. Analyze the limits involving infinity graphically, numerically, algebraically, and in context.	1. Explain how and why the concept of limits and horizontal asymptotes are related. 2. Explain how and why the concept of limits and vertical asymptotes are related. 3. Identify infinite limits given the graph of a function or the equation of a function 4. Identify limits at infinity given the graph of a function or the equation of a function 5. Sketch the graph of a function given information about infinite limits and limits at infinity 6. Find a formula for a function given horizontal and vertical asymptotes	*Calculus: Concepts & Contexts Section 2.5* District constructed materials	1. Make sense of problems and persevere in solving them. 2. Reason abstractly and quantitatively. 3. Construct viable arguments and critique the reasoning of others. 4. Model with math. 5. Use appropriate tools strategically. 6. Attend to precision. 7. Look for and make use of structure. 8. Look for and express regularity in repeated reasoning.
4. Represent and relate problems involving tangents, velocities and other rates of change	1. Find the slope of the secant line between two points and apply that slope to a line tangent to the curve at a specific point. 2. Find the slope of the tangent line drawn to a curve at a given point using the limit definition. 3. Find the equation of the tangent line drawn to a curve at a given point and interpret its meaning. 4. Explain how and why the concept of limits and finding the slope of a tangent line to a given curve are related. 5. Find the velocity of a moving particle at a specific point given its position function 6. Sketch a graph of the position of a moving particle given information about its velocity and a starting point	*Calculus: Concepts & Contexts* Section 2.6 http://www.calculus-help.com/tutorials District constructed materials	1. Make sense of problems and persevere in solving them. 2. Reason abstractly and quantitatively. 3. Construct viable arguments and critique the reasoning of others. 4. Model with math. 5. Use appropriate tools strategically. 6. Attend to precision. 7. Look for and make use of structure. 8. Look for and express regularity in repeated reasoning.

Learning Objective The student will…	Content Outline	Instructional Materials	Standards for Mathematical Practice
5. Interpret and represent the derivative of a function.	1. Model the geometric interpretation of a derivative. 2. Evaluate the derivative of a function at a given point and interpret its geometric meaning. 3. Find the derivative (f ' (c)) given the graph of equation of the function (f(x))	*Calculus: Concepts & Contexts* *Section 2.7* http://www.calculus-help.com/tutorials District constructed materials	1. Make sense of problems and persevere in solving them. 2. Reason abstractly and quantitatively. 3. Construct viable arguments and critique the reasoning of others. 4. Model with math. 5. Use appropriate tools strategically. 6. Attend to precision. 7. Look for and make use of structure. 8. Look for and express regularity in repeated reasoning.
6. Analyze graphs of function and its first and second derivatives and distinguish characteristics of each.	1. Investigate and generalize the relationship between the graph of f and f'. 2. Investigate and generalize the relationship between the graph of f' and f'''' 3. Given the graph of a function estimate the derivative at various given points. 4. Given the graph of a function, create a sketch of its derivative function 5. Given an equation for a function (f(x)), find the derivative (f ' (x)) algebraically 6. Given a sketch of a function, identify where it is not differentiable and why 7. Given a sketch of a function and its first and second derivatives, distinguish between the two graphs.	*Calculus: Concepts & Contexts* Section 2.8 http://www.calculus-help.com/tutorials District constructed materials	1. Make sense of problems and persevere in solving them. 2. Reason abstractly and quantitatively. 3. Construct viable arguments and critique the reasoning of others. 4. Model with math. 5. Use appropriate tools strategically. 6. Attend to precision. 7. Look for and make use of structure. 8. Look for and express regularity in repeated reasoning.

Learning Objective The student will…	Content Outline	Instructional Materials	Standards for Mathematical Practice
7a. Connect and apply the antiderivative to the derivative 7b. Interpret graphs of the derivative of a function	1. Investigate how derivatives and antiderivatives related to each other graphically 2. Given the graph of a derivative function, identify the places where the function is increasing/decreasing, concave up/down, local max/min, points of inflection 3. Sketch a graph of a function given information about its derivative 4. Determine and explain whether or not there is a one-to-one relationship between derivatives and antiderivatives 5. Sketch a graph of the antiderivative of a given function	*Calculus: Concepts & Contexts* *Section 2.9* http://www.calculus-help.com/tutorials District constructed materials	1. Make sense of problems and persevere in solving them. 2. Reason abstractly and quantitatively. 3. Construct viable arguments and critique the reasoning of others. 4. Model with math. 5. Use appropriate tools strategically. 6. Attend to precision. 7. Look for and make use of structure. 8. Look for and express regularity in repeated reasoning.
8. Represent and evaluate problems involving derivatives of polynomials and exponential functions	1. Find derivatives of a constant function, a power function, the product of a constant and a function, a sum/difference of functions and an exponential function. 2. Write the equation of the tangents and normal lines to these functions at specific points. 3. Find higher order derivatives.	*Calculus: Concepts & Contexts* Section 3.1 http://www.calculus-help.com/tutorials District constructed materials	1. Make sense of problems and persevere in solving them. 2. Reason abstractly and quantitatively. 3. Construct viable arguments and critique the reasoning of others. 4. Model with math. 5. Use appropriate tools strategically. 6. Attend to precision. 7. Look for and make use of structure. 8. Look for and express regularity in repeated reasoning.

Learning Objective The student will…	Content Outline	Instructional Materials	Standards for Mathematical Practice
9. Interpret and apply the product and quotient rules.	1. Determine and justify if the derivative of the product of two functions equal to the product of the derivative of each function 2. Find derivatives functions that are the product/quotient of two differentiable functions.	*Calculus: Concepts & Contexts* Section 3.2 District constructed materials	1. Make sense of problems and persevere in solving them. 2. Reason abstractly and quantitatively. 3. Construct viable arguments and critique the reasoning of others. 4. Model with math. 5. Use appropriate tools strategically. 6. Attend to precision. 7. Look for and make use of structure. 8. Look for and express regularity in repeated reasoning.
10. Apply rates of change to physics and economics.	1. Interpret real applications, such as If f(t) represents the monthly revenue in dollars and t is measured in months, what is the meaning of f'(3) = 100? 2. Find and interpret the derivative given an initial function.	*Calculus: Concepts & Contexts Section 3.3* District constructed materials	1. Make sense of problems and persevere in solving them. 2. Reason abstractly and quantitatively. 3. Construct viable arguments and critique the reasoning of others. 4. Model with math. 5. Use appropriate tools strategically. 6. Attend to precision. 7. Look for and make use of structure. 8. Look for and express regularity in repeated reasoning.

Learning Objective The student will…	Content Outline	Instructional Materials	Standards for Mathematical Practice
11. Investigate and evaluate the derivative of the sine and cosine function.	1. Use the graphs of the sine and cosine functions to prove their derivatives. 2. Evaluate the derivative of the sine and cosine function at given points.	*Calculus: Concepts & Contexts* Section 3.4 District constructed materials	1. Make sense of problems and persevere in solving them. 2. Reason abstractly and quantitatively. 3. Construct viable arguments and critique the reasoning of others. 4. Model with math. 5. Use appropriate tools strategically. 6. Attend to precision. 7. Look for and make use of structure. 8. Look for and express regularity in repeated reasoning.
12. Apply the chain rule to find and evaluate derivatives of composite functions.	1. Identify a complicated function as the composition of two or more functions and identify the order. 2. Analyze how and why the chain rule works, 3. Implement the chain rule when asked to find the derivative of a composite function. 4. Relate and connect the chain rule to appropriate functions. 5. Apply other methods of differentiation when the chain rule is not necessary.	*Calculus: Concepts & Contexts* Section 3.5 District constructed materials	1. Make sense of problems and persevere in solving them. 2. Reason abstractly and quantitatively. 3. Construct viable arguments and critique the reasoning of others. 4. Model with math. 5. Use appropriate tools strategically. 6. Attend to precision. 7. Look for and make use of structure. 8. Look for and express regularity in repeated reasoning.

Learning Objective The student will…	Content Outline	Instructional Materials	Standards for Mathematical Practice
13. Implement implicit differentiation.	1. Relate the chain rule to implicit differentiation. 2. Distinguish functions expressed implicitly and explicitly. 3. Initiate the use of implicit differentiation when appropriate.	*Calculus: Concepts & Contexts* Section 3.6 District constructed materials	1. Make sense of problems and persevere in solving them. 2. Reason abstractly and quantitatively. 3. Construct viable arguments and critique the reasoning of others. 4. Model with math. 5. Use appropriate tools strategically. 6. Attend to precision. 7. Look for and make use of structure. 8. Look for and express regularity in repeated reasoning.
14. Apply the derivative as a rate of change to interpret real world problems.	1. Formulate a mathematical model of a real world problem relating the given and unknown rates of change. 2. Differentiate both sides of an equation implicitly with respect to time. 3. Interpret a solution by translating the mathematical result in context.	*Calculus: Concepts & Contexts* Section 4.1 District constructed materials	1. Make sense of problems and persevere in solving them. 2. Reason abstractly and quantitatively. 3. Construct viable arguments and critique the reasoning of others. 4. Model with math. 5. Use appropriate tools strategically. 6. Attend to precision. 7. Look for and make use of structure. 8. Look for and express regularity in repeated reasoning.
15. Connect and relate the antiderivative to the derivative.	1. Find general antiderivatives and determine if every function has a unique antiderivative.	*Calculus: Concepts & Contexts* Section 4.9 District constructed materials	1. Make sense of problems and persevere in solving them. 2. Reason abstractly and quantitatively. 3. Construct viable arguments and critique the reasoning of others. 4. Model with math. 5. Use appropriate tools strategically. 6. Attend to precision. 7. Look for and make use of structure.

			8. Look for and express regularity in repeated reasoning.

South Orange & Maplewood School District
Mathematics Grades 11-12
Topics in Calculus

Recommended Unit Sequencing and Pacing Guide

Timeframe	Topics
Q3	Limits & Derivatives The Tangent & Velocity Problems (2.1) The Limit of a Function (2.2) Continuity (2.4) Limits Involving Infinity (2.5) Tangents, Velocity and Other Rates of Change (2.6) Derivatives (2.7) The Derivative As A Function (2.8) What Does f ' Say About f? (2.9)
Q3	Differentiation Rules Derivatives of Polynomial & Exponential Functions (3.1) The Product & Quotient Rules (3.2) Rates of Change in Physics and Economics (3.3) Derivatives of Sine and Cosine Functions (3.4) The Chain Rule (3.5) Implicit Differentiation (3.6)
Q4	Applications of Differentiation Related Rates (4.1) Maximum & Minimum Values (4.2) Derivatives & The Shape of Curves of Polynomials (4.3) Graphing With Calculus & Calculators (4.4) Optimization Problems (4.6) Antiderivatives (4.9)
	Final Exam