

South Orange & Maplewood School District Mathematics Curriculum Grade 11-12 Trigonometry



South Orange Maplewood
School District
Department of Curriculum &
Instruction
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South Orange & Maplewood School District
Mathematics Grades 11-12
Trigonometry
TABLE OF CONTENTS

Section I	1
South Orange & Maplewood Board of Education.....	1
Acknowledgements.....	1
Course Description.....	2
Evaluation.....	2
Common Core State Standards.....	2
Textbook.....	2
Section II	3
Trigonometry Curriculum.....	3
Learning Objectives.....	3
Pacing.....	3
Content Outline.....	3
Instructional Materials.....	3
Notes.....	3
Standards of Mathematical Practice.....	3

School District of South Orange and Maplewood
Trigonometry

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ACKNOWLEDGEMENTS

Curriculum Writers

Candice Beattys
Megan Power
Elaine Weiland

School District of South Orange and Maplewood

Trigonometry

Course Title: Trigonometry (Level 3 or Level 4)

Grade: 11 or 12

Length of Course: One Summer Semester (5.0 credits)

Prerequisites: Algebra 2 (Level 3) or Algebra II Honors (Level 4)

Description:

This Trigonometry course aims at preparing students for success in a Precalculus approach to the study of functions and Topics in Calculus. Introducing and connecting periodic functions to motion and real world phenomena are important topics and involve higher level thinking. Circular functions are mathematical models for real life analysis such as the measurement of blood pressure and heart beats, analyzing tide variations and chemotherapy analysis to offer some examples. To succeed in any Calculus class, students must have a firm understanding of triangle trigonometry, trigonometric functions, and analytic trigonometry.

This course is intended to prepare students for their college mathematics classes. It is designed to provide a balance between the development of key concepts and the mastery of skills. With that in mind, this curriculum clearly defines the learning objectives for each unit in terms of the key concepts and essential skills that must be mastered within each unit.

Evaluation:

Student performance will be measured using a variety of instructor-specific quizzes and chapter tests as well as a mid-semester exam and final exam. Assessments will equally emphasize measurement of the degree to which required skills have been mastered as well as how well concepts have been understood.

Scope and Sequence:

This is a multilevel course designed for instruction at Level 3 and Level 4. A pacing guide is attached.

Text:

Level 4: *Precalculus Mathematics for Calculus 5th Edition*, James Stewart, Lothar Redlin and Saleem Watson, Thomson Brooks/Cole 2006 [Stewart]

Reference Texts:

Advanced Mathematics, Precalculus with Discrete Mathematics and Data Analysis, Richard G. Brown, Houghton Mifflin Company 1992 [Brown]

School District of South Orange and Maplewood

Trigonometry

Precalculus with Trigonometry, Concepts and Applications, Paul A. Foerster, Key Curriculum Press 2003 [Foerster]

Precalculus with Limits: A Graphical Approach 4th Edition, Larson, McDougal Littell [Larson]

School District of South Orange and Maplewood
Trigonometry

Unit 1: Functions, Inverses and Their Graphs

Learning Objectives	Content Outline Skills and Concepts	Instructional Materials	Standards for Mathematical Practice
<p>1a. Analyze and relate attributes of functions graphically, symbolically and in context.</p> <p>1b. Connect selected parent functions to their transformations symbolically and graphically.</p> <p>CCSS:</p>	<ol style="list-style-type: none"> 1. Analyze and relate attributes of functions graphically, symbolically, and in context. 2. Construct selected parent function graphs and interpret transformations 3. Apply function transformations to their graphs and their equations 4. Apply concept of symmetry to selected functions 5. Analyze and interpret intervals where functions are increasing and decreasing and representations in context 	<p>[Stewart] Section 2.4</p>	<ol style="list-style-type: none"> 1. Make sense of problems and persevere in solving them. 2. Reason abstractly and quantitatively. 3. Construct viable arguments and critique the reasoning of others. 4. <u>Model with math.</u> 5. <u>Use appropriate tools strategically.</u> 6. Attend to precision. 7. <u>Look for and make use of structure.</u> 8. <u>Look for and express regularity in repeated reasoning.</u>
<p>2. Recognize interpret and model inverse of selected functions algebraically, graphically and in context of real world examples</p> <p>CCSS:</p>	<ol style="list-style-type: none"> 1. Apply concept of graphical symmetry to functions and their inverses. 2. Construct an inverse given a function and interpret results 3. Analyze properties of inverse functions and verify if a function exists or if it is a one to one function 	<p>[Stewart] Section 2.8</p>	<ol style="list-style-type: none"> 1. <u>Make sense of problems and persevere in solving them.</u> 2. Reason abstractly and quantitatively. 3. <u>Construct viable arguments and critique the reasoning of others.</u> 4. Model with math. 5. Use appropriate tools strategically. 6. Attend to precision. 7. Look for and make use of structure. 8. <u>Look for and express regularity in repeated reasoning.</u>

School District of South Orange and Maplewood
Trigonometry

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<p>3. Explore and interpret composition of functions and its effect on the domain and range of the newly composed function.</p> <p>CCSS:</p>	<ol style="list-style-type: none"> 1. Formulate the composition of two functions. 2. Investigate the restrictions on domain when a function input is restricted by another function 3. Apply algebraic reasoning of compositions to the composition of a function and its inverse in determining if the function is one to one. 4. Connect and analyze the impact the composing functions that has a domain restriction that continues to impact the newly composed function, including composing inverse functions. 	<p>[Stewart] Section 2.7</p>	<ol style="list-style-type: none"> 1. <u>Make sense of problems and persevere in solving them</u> . 2. Reason abstractly and quantitatively. 3. Construct viable arguments and critique the reasoning of others. 4. Model with math. 5. Use appropriate tools strategically 6. <u>Attend to precision</u>. 7. Look for and make use of structure. 8. Look for and express regularity in repeated reasoning.

School District of South Orange and Maplewood
Trigonometry

Unit 2: Trigonometric Ratios

Learning Objective	Content Outline Skills and Concepts	Instructional Materials	Standards for Mathematical Practice
<p>4. Investigate and apply trigonometric ratios.</p> <p>CCSS: G.SRT.C.6 G.SRT.C.8 F.TF.A.3</p>	<ol style="list-style-type: none"> 1. Review the concept of triangle similarity and the application to all triangles. 2. Interpret and evaluate the six trigonometric ratios of special right triangles. 3. Investigate and explore how sine, cosine and tangent ratios are defined in a right triangle. 4. Connect the proportionality of the sides of any triangle to the trigonometric angle measure. 5. Evaluate and analyze exact values of trigonometric ratios when given two lengths of a right triangle. 6. Interpret right triangles by solving using trigonometric functions. 7. Apply knowledge of the Pythagorean Theorem and altitude to finding trigonometric values of special right triangles (45-45-90) and (30-60-90) without the use of a calculator. 8. Integrate the application of the inverse function on the calculator to solve for angles in application problems. 9. Justify why trigonometric ratios within a right triangle make sense using the geometric theorem of similarity. 	<p>[Stewart] Section 6.2 (Trigonometry of Right Triangles)</p>	<ol style="list-style-type: none"> 1. Make sense of problems and persevere in solving them . 2. Reason abstractly and quantitatively. 3. <u>Construct viable arguments and critique the reasoning of others.</u> 4. <u>Model with math.</u> 5. Use appropriate tools strategically 6. Attend to precision. 7. Look for and make use of structure. 8. Look for and express regularity in repeated reasoning.

School District of South Orange and Maplewood
Trigonometry

Learning Objective	Content Outline Skills and Concepts	Instructional Materials	Standards for Mathematical Practice
<p>5. Represent and interpret the measurement of radians to the radius of each circle encompassing the circumference 2π.</p> <p>CCSS: F.TF.A.1 F.TF.A.2</p>	<ol style="list-style-type: none"> 1. Apply radian measure to the fractional arc of a circle and equate it the central angle it intercepts, and rotations completed 2. Analyze the relationship between degree and radians and use either measurement in context. Create a proportion to evaluate radians and degrees. 3. Distinguish radian measure from degree measure of angles and explain conceptual understanding of radian measure (relationship to circumference of a unit circle). 4. Convert between degree and radian measure. 5. Interpret a positive and negative angle in standard position and connect the concept of coterminal angles and rotation counterclockwise or clockwise respectively. 6. Find arc length and sector areas 7. Derive the formulas for the arc length and sector area when using an angle in radian measure. 	<p>[Stewart] Section 6.1 (Angle Measure)</p>	<ol style="list-style-type: none"> 1. Make sense of problems and persevere in solving them . 2. <u>Reason abstractly and quantitatively.</u> 3. <u>Construct viable arguments and critique the reasoning of others.</u> 4. <u>Model with math.</u> 5. Use appropriate tools strategically 6. Attend to precision. 7. <u>Look for and make use of structure.</u> 8. <u>Look for and express regularity in repeated reasoning.</u>

School District of South Orange and Maplewood Trigonometry

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<p>6. Identify and interpret the value of a trigonometric function of an angle of any size</p> <p>CCSS F.TF.A.2 F.TF.A.3</p>	<ol style="list-style-type: none"> 1. Construct and apply the six trigonometric ratios when the angle is placed in standard position. 2. Investigate reference angles through the construction of right triangles within the unit circle for any angle in standard position. 3. Interpret and distinguish reference angles and angles in standard position. 4. Connect and analyze the quadrant where the terminal side of an angle must lie given the signs of the trigonometric functions 5. Evaluate the exact values of the remaining trig functions when given one or more of its trig ratios. 6. Connect and verify that an angle in any quadrant with equivalent reference angles have the same trigonometric ratio values adjusted for sign (+/-). 	<p>[Stewart] Section 6.3 (Trigonometric Functions of Angles)</p>	<ol style="list-style-type: none"> 1. Make sense of problems and persevere in solving them . 2. Reason abstractly and quantitatively. 3. <u>Construct viable arguments and critique the reasoning of others.</u> 4. Model with math. 5. Use appropriate tools strategically 6. <u>Attend to precision.</u> 7. <u>Look for and make use of structure.</u> 8. Look for and express regularity in repeated reasoning.

School District of South Orange and Maplewood
Trigonometry

Learning Objective	Content Outline Skills and Concepts	Instructional Materials	Standards for Mathematical Practice
<p>7. Develop the model of the unit circle to find the trigonometric ratios of a given angle (of any size)</p> <p>CCSS: F.TF.C.8 F.TF.A.4</p>	<ol style="list-style-type: none"> 1. Represent and define the unit circle as a circle with radius of one. 2. Relate the unit circle and special right triangles to find the values of the six trigonometric ratios for special angles in any quadrant. 3. Investigate the terminal point of a given rotation around the unit circle and relate it as the coordinate on the Cartesian plane. 4. Connect the terminal point of an angle on the unit circle to its related trigonometric functions. 5. Illustrate the relationship by creating the right triangle on the unit circle (with reference to the x-axis) – highlighting cosine represents x-values and sine represents y-values. 	<p>[Stewart] Section 5.1 (The Unit Circle), [Stewart] 5.2 (The Trigonometric Functions of Real Numbers)</p>	<ol style="list-style-type: none"> 1. <u>Make sense of problems and persevere in solving them .</u> 2. Reason abstractly and quantitatively. 3. <u>Construct viable arguments and critique the reasoning of others.</u> 4. Model with math. 5. Use appropriate tools strategically 6. Attend to precision. 7. <u>Look for and make use of structure.</u> 8. <u>Look for and express regularity in repeated reasoning.</u>

School District of South Orange and Maplewood
Trigonometry

Unit 3 : Trigonometric Functions

Learning Objectives	Content Outline Skills and Concepts	Instructional Materials	Standards for Mathematical Practice
<p>8. Recognize and sketch graphs of trigonometric functions and identify their key attributes.</p> <p>CCSS: F.TF.A.4</p>	<ol style="list-style-type: none"> 1. Construct a periodic graph. Interpret its unique attributes of amplitude, period and phase shift. 2. Explain why trigonometric functions are periodic. Explain why some trigonometric graphs have asymptotes and identify where the asymptotes occur. 3. Determine domain and range of the six trigonometric graphs. 4. Recognize the graphs of the six trigonometric functions, with emphasis on sine and cosine functions. 5. Apply transformations to graph sine and cosine functions. 6. State amplitude, period, phase shift, domain, and range when given an equation. 7. Create an equation when given transformation descriptions or a graph. 	<p>[Stewart] 5.3 (Trigonometric Graphs), [Stewart] 5.4 (More Trigonometric Graphs)</p> <p>“Graphs from the Unit Circle” http://illuminations.nctm.org/ LessonDetail.aspx?id=L785</p> <p>“Tidal Waves,” <u>Focus in High School Mathematics.</u></p>	<ol style="list-style-type: none"> 1. <u>Make sense of problems and persevere in solving them .</u> 2. Reason abstractly and quantitatively. 3. Construct viable arguments and critique the reasoning of others. 4. <u>Model with math.</u> 5. <u>Use appropriate tools strategically</u> 6. Attend to precision. 7. Look for and make use of structure. 8. <u>Look for and express regularity in repeated reasoning.</u>

School District of South Orange and Maplewood
Trigonometry

Learning Objectives	Content Outline Skills and Concepts	Instructional Materials	Standards for Mathematical Practice
<p>9a. Investigate and connect the concept of inverse functions as it relates to trigonometric functions.</p> <p>9b. Analyze and evaluate inverse trigonometric functions.</p> <p>9c. Explore and analyze compositions of trigonometric functions and inverse trigonometric functions.</p> <p>CCSS: F.TF.B.7</p>	<ol style="list-style-type: none"> 1. Construct the inverse applying properties of functions to restrict its domain and range. 2. Connect domain and range restrictions in order to evaluate inverse of the sine, cosine, and tangent. 3. Apply reciprocals function to analyze and evaluate inverse cosecant, secant, and cotangent. 4. Explore the use of inverse trigonometric functions on the calculator to evaluate simple trigonometric equations that do not involve special angle measures or special side ratios. 	<p>[Stewart] Section 7.4 (Inverse Trigonometric Functions)</p>	<ol style="list-style-type: none"> 1. <u>Make sense of problems and persevere in solving them .</u> 2. <u>Reason abstractly and quantitatively.</u> 3. Construct viable arguments and critique the reasoning of others. 4. Model with math. 5. <u>Use appropriate tools strategically</u> 6. <u>Attend to precision.</u> 7. Look for and make use of structure. 8. Look for and express regularity in repeated reasoning.

School District of South Orange and Maplewood
Trigonometry

Unit 4 : Analytic Trigonometry and Applications of Trigonometric Functions

Learning Objectives	Content Outline Key Definitions, Skills and Concepts	Instructional Materials	Standards for Mathematical Practice
<p>10a. Recognize and verify trigonometric identities</p> <p>10b. Implement the use of trigonometric identities to simplify trigonometric expressions</p> <p>CCSS: F.TF.C.8</p>	<ol style="list-style-type: none"> 1. Distinguish and analyze the difference between an identity and an equation. 2. Derive reciprocal identities and Pythagorean identities. 3. Recognize and create all forms of each Pythagorean identity. 4. Formulate and investigate the relationship between the equation of any circle, the Pythagorean theorem, and Pythagorean identities. 5. Implement the use trigonometric in simplifying trigonometric expressions. 6. Implement the use of known algebraic methods in conjunction with trigonometric identities to verify a trigonometric equation is an identity. 	<p>[Stewart] Section 7.1 (Trigonometric Identities)</p>	<ol style="list-style-type: none"> 1. <u>Make sense of problems and persevere in solving them .</u> 2. <u>Reason abstractly and quantitatively.</u> 3. <u>Construct viable arguments and critique the reasoning of others.</u> 4. <u>Model with math.</u> 5. Use appropriate tools strategically 6. Attend to precision. 7. <u>Look for and make use of structure.</u> 8. <u>Look for and express regularity in repeated reasoning.</u>

School District of South Orange and Maplewood Trigonometry

Learning Objectives	Content Outline Key Definitions, Skills and Concepts	Instructional Materials	Standards for Mathematical Practice
<p>11. Integrate the study of trigonometry and the unit circle to find all solutions to trigonometric equations</p> <p>CCSS: F.TF.B.7</p>	<ol style="list-style-type: none"> 1. Isolate a trigonometric function by applying algebraic skills and integrating the concept of a periodic function to find all solutions to an equation. 2. Connect and recognize when a trigonometric equation has a quadratic form and link with knowledge of solving quadratics. 3. Formulate solutions to trigonometric equations when using inverse trigonometric functions and incorporate all solutions as it rotates around the unit circle. 	<p>[Stewart] Section 7.5 (Trigonometric Equations)</p>	<ol style="list-style-type: none"> 1. <u>Make sense of problems and persevere in solving them .</u> 2. <u>Reason abstractly and quantitatively.</u> 3. <u>Construct viable arguments and critique the reasoning of others.</u> 4. <u>Model with math.</u> 5. Use appropriate tools strategically 6. <u>Attend to precision.</u> 7. Look for and make use of structure. 8. <u>Look for and express regularity in repeated reasoning.</u>
<p>12. Use the angle addition and angle subtraction formulas for sine, cosine and tangent to evaluate trigonometric equations.</p> <p>CCSS: F.TF.C.9</p>	<ol style="list-style-type: none"> 1. Verify and investigate the angle addition and subtraction formulas for sine, cosine and tangent. Apply the angle addition and subtraction formulas to evaluate trigonometric expressions. 2. Derive the double angle formulas for sine, cosine, and tangent. 	<p>[Stewart] Section 7.2 (Addition and Subtraction Formulas)</p>	<ol style="list-style-type: none"> 1. Make sense of problems and persevere in solving them . 2. Reason abstractly and quantitatively. 3. <u>Construct viable arguments and critique the reasoning of others.</u> 4. Model with math. 5. Use appropriate tools strategically 6. Attend to precision. 7. <u>Look for and make use of structure.</u> 8. <u>Look for and express regularity in repeated reasoning.</u>

School District of South Orange and Maplewood Trigonometry

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<p>13. Develop and apply the Law of Sines and the Law of Cosines to solve triangles and triangle applications.</p> <p>CCSS: G.SRT.D.10 G.SRT.D.11</p>	<ol style="list-style-type: none"> 1. Investigate the use of the Law of Sines and the Law of Cosines. 2. Apply the Law of Sines to completion when presented with ambiguous case. Relate the ambiguous case to the sine function and its symmetry across the y-axis. 3. Implement the Law of Sines and the Law of Cosines to solve for all possible triangles when given a set of conditions. 4. Distinguish when to use Law of Sines or Law of Cosines when given specific information about a triangle . 	<p>12-4: [Stewart] 6.4 (The Law of Sines), [Stewart] 6.5 (The Law of Cosines)</p> <p><u>The Roles of Representation in School of Mathematics:</u> "A Multiple Representation Journey through Law of Cosines, "</p>	<ol style="list-style-type: none"> 1. Make sense of problems and persevere in solving them . 2. Reason abstractly and quantitatively. 3. <u>Construct viable arguments and critique the reasoning of others.</u> 4. <u>Model with math.</u> 5. <u>Use appropriate tools strategically</u> 6. Attend to precision. 7. Look for and make use of structure. 8. Look for and express regularity in repeated reasoning.
<p>14. Interpret real world data to model Sinusoidal Functions as Mathematically Models</p> <p>CCSS: F.TF.B.5 F.TF.B.7</p>	<ol style="list-style-type: none"> 1. Interpret real world data to form a trigonometric mathematical model. 2. Apply this model to answer questions about the data 3. Analyze this model to make reasonable predictions about the future. 	<p>12-4: [Stewart] 5.5 (Modeling Harmonic Motion)</p> <p><u>Mathematic Assessment Sampler:</u> "Ferris Wheel Problem",</p>	<ol style="list-style-type: none"> 1. <u>Make sense of problems and persevere in solving them .</u> 2. <u>Reason abstractly and quantitatively.</u> 3. Construct viable arguments and critique the reasoning of others. 4. <u>Model with math.</u> 5. <u>Use appropriate tools strategically</u> 6. <u>Attend to precision.</u> 7. Look for and make use of structure. 8. Look for and express regularity in repeated reasoning.