

# **South Orange & Maplewood School District Mathematics Curriculum Grade 11-12 Pre-Calculus and Functions Analysis**



**South Orange Maplewood  
School District  
Department of Curriculum &  
Instruction  
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South Orange & Maplewood School District  
Mathematics Grade 11-12  
Pre-Calculus and Functions Analysis

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South Orange & Maplewood School District  
Mathematics Grade 11-12  
Pre-Calculus and Functions Analysis

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South Orange & Maplewood School District  
Mathematics Grade 11-12  
Pre-Calculus and Functions Analysis

**Course Title:** Pre-calculus and Function Analysis (Level 3 and Level 4)

**Length of Course:** One semester (2.5 credits)

**Prerequisite:** Trigonometry

**Description:**

This Precalculus course generally centers on functions, and specifically aims to enable students to decide just what kind of function will best fit a set of real-world data that is inherently scattered. Applications to real world problems sometimes initiate or motive a topic; other times, the big picture is only unveiled after various parts come together to reveal the mathematical structure. Fundamentally, this course evidences the importance of mathematics to a wide variety of other disciplines; the more a student understands mathematics, the more deeply they will understand other subjects.

This course is designed to expand students' knowledge of functions to include: using functions as mathematical models; relating transformations that can be performed on functions; and generalizing properties and behaviors that apply to a variety of functions, including polynomial, exponential, logarithmic and rational functions. As they did in Algebra 2, students will regularly work with functions in four ways: applying mathematics in real situations; representing mathematical concepts algebraically, verbally, numerically, and graphically; developing and strategizing through mathematical properties and relationships; and carrying out various algorithms.

This course strives to give students a balance between the acquisition of key concepts and the mastery of skills that lay the foundation for calculus. To map this course, this curriculum guide clearly defines the learning objectives and a specific content outline for each objective.

**Evaluation:**

The distinction between levels will be based on the range and mastery of content. Every assessment will provide a core set of concepts, skills and applications, as well as a variety of enriched or challenge problems to evidence mastery at an Honors level. Student performance will be measured using a variety of quizzes and chapter tests as well as departmental common assessments, and a Final Exam.

**Text:** *PreCalculus Mathematics for Calculus, 5<sup>th</sup> Edition*, James Stewart, Lothar Redlin, and Saleem Watson, Thomson Brooks/Cole 2006 [Stewart].

**Reference Texts:**

*Functions, Statistics and Trigonometry*, Rubenstein, Schultz, et al. Scott, Foresman and Company 1992 [FST]

*Advanced Mathematics, Precalculus with Discrete Mathematics and Data Analysis*, Richard G. Brown, Houghton Mifflin Company 1992 [Brown]

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Mathematics Grade 11-12

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*Precalculus with Trigonometry, Concepts and Applications*, Paul A. Foerster, Key Curriculum Press 2003 [Foerster]

*Precalculus with Limits: A Graphical Approach 4<sup>th</sup> Edition*, Larson, McDougal Littell [Larso]

# Precalculus and Function Analysis

Objective	Content Outline Skills and Concepts	Instructional Materials	Standards for Mathematical Practice
<p>1a. Recognize or generate equivalent forms for exponential, radical, and polynomial expressions.</p> <p>1b. Apply algebraic strategies to exponential radical, rational and polynomial expressions.</p>	<ol style="list-style-type: none"> <li>Investigate the connection between fractional exponents, roots and powers of whole numbers.</li> <li>Simplify expressions with positive, negative and fractional exponents.</li> <li>Simplify expressions with roots of degree 2 or higher.</li> <li>Convert between radical notation and fractional exponent. .</li> <li>Factor polynomials using a variety of techniques, such as factoring formulas, application of known multiples or factoring a common monomial.</li> <li>Apply the concept of a polynomial common denominator and use common denominational properties to perform operations on rational expressions.</li> <li>Utilize the conjugate to rationalize a radical expression.</li> <li>Solve multivariate equations for a given variable through use of advanced algebraic techniques.</li> </ol>	<p>Stewart: Section 1.2 (Exponents and Radicals), Section 1.3 (Algebraic Expression) Section 1.4 (Rational Expressions)</p>	<ol style="list-style-type: none"> <li>Make sense of problems and persevere in solving them.</li> <li>Reason abstractly and quantitatively.</li> <li><u>Construct viable arguments and critique the reasoning of others.</u></li> <li>Model with math.</li> <li>Use appropriate tools strategically.</li> <li>Attend to precision.</li> <li><u>Look for and make use of structure.</u></li> <li><u>Look for and express regularity in repeated reasoning.</u></li> </ol>
<p>2. Distinguish, select, apply, and explain a particular method of solving a quadratic based on the graph or the form of the equation</p>	<ol style="list-style-type: none"> <li>Select a method and solve quadratic equations by factoring, completing the square or the quadratic formula</li> <li>Connect and apply the roots as zeros of the functions and x-intercepts of the graph.</li> <li>Solve polynomials functions using the strategy factoring by grouping.</li> </ol>	<p>Stewart: Section 1.5 (Equations)</p>	<ol style="list-style-type: none"> <li><u>Make sense of problems and persevere in solving them.</u></li> <li>Reason abstractly and quantitatively.</li> <li>Construct viable arguments and critique the reasoning of others.</li> <li>Model with math.</li> <li><u>Use appropriate tools strategically.</u></li> <li><u>Attend to precision.</u></li> <li><u>Look for and make use of structure.</u></li> <li><u>Look for and express regularity in repeated reasoning.</u></li> </ol>

# Precalculus and Function Analysis

Objective	Content Outline Skills and Concepts	Instructional Materials	Standards for Mathematical Practice
3. Interpret and analyze attributes of a function and its domain and range.	<ol style="list-style-type: none"> <li>1. Analyze a graph to identify a function, interpret intervals where the function increases and decreases, and identify domain and range.</li> <li>2. Calculate the average rate of change of a function and interpret its meaning.</li> <li>3. Interpret average rate of change as used in real world analysis (i.e. climate change, water level changes, population growth/decline)</li> <li>4. Interpret a function's equation for its domain and range.</li> <li>5. Interpret a piece-wise function's equation for domain and range.</li> <li>6. Construct the graph of a piece-wise function.</li> <li>7. Explore real world applications of quadratic and polynomial functions through analysis of maxima and the minima.</li> </ol>	<p>Stewart: Section 2.1 (What is a Function?) Section 2.2 (Graphs of Functions) Section 2.3 (Increasing and Decreasing Functions) Section 2.6 (Modeling with Functions)</p>	<ol style="list-style-type: none"> <li>1. <u>Make sense of problems and persevere in solving them.</u></li> <li>2. <u>Reason abstractly and quantitatively.</u></li> <li>3. <u>Construct viable arguments and critique the reasoning of others.</u></li> <li>4. Model with math.</li> <li>5. <u>Use appropriate tools strategically.</u></li> <li>6. <u>Attend to precision.</u></li> <li>7. Look for and make use of structure.</li> <li>8. Look for and express regularity in repeated reasoning.</li> </ol>
4. Combine functions to create new functions and identify their resulting domains.	<ol style="list-style-type: none"> <li>1. Apply mathematical operations to the combination of two or more functions and identify their resulting domains.</li> <li>2. Decompose a given function into a composition expression.</li> </ol>	<p>Stewart: Section 2.7 (Combining Functions)</p>	<ol style="list-style-type: none"> <li>1. <u>Make sense of problems and persevere in solving them.</u></li> <li>2. Reason abstractly and quantitatively.</li> <li>3. Construct viable arguments and critique the reasoning of others.</li> <li>4. Model with math.</li> <li>5. <u>Use appropriate tools strategically.</u></li> <li>6. <u>Attend to precision.</u></li> <li>7. <u>Look for and make use of structure.</u></li> <li>8. Look for and express regularity in repeated reasoning.</li> </ol>

# Precalculus and Function Analysis

Objective	Content Outline Skills and Concepts	Instructional Materials	Standards for Mathematical Practice
<p>5. Represent, interpret and solve problems involving polynomial functions.</p> <p>6. Identify the key attributes of a polynomial function and its graph.</p> <p>7. Apply the Fundamental theorem of Algebra to finding all the zeros of a polynomial real and complex.</p>	<ol style="list-style-type: none"> <li>1. Interpret graphs of polynomials for its unique attributes of end behavior, roots of the equation (real and complex), and behavior near the zeros of the function.</li> <li>2. Evaluate and analyze polynomial functions for its zeros by interpreting the graph, applying algebraic techniques, and implementing the Remainder Theorem.</li> <li>3. Analyze the graphs of polynomial for extrema and multiplicity.</li> <li>4. Explain why complex zeros come in pairs.</li> <li>5. Use the complex conjugate theorem to create polynomials given one complex root.</li> <li>6. Find all zeros of a given polynomial including the real and complex roots.</li> <li>7. Interpret the meaning of the zeros of a polynomial and explain what they tell us about the function.</li> <li>8. Connect the zeros/roots of a function to the factors of the polynomials. Find all zeroes of a given polynomial including the real and complex roots.</li> </ol>	<p>Stewart: Section 3.1 (Polynomial Functions and Their Graphs) Section 3.2 (Dividing Polynomials) Section 3.3 (Real Zeros of Polynomials) Section 3.4 (Complex Numbers) Section 3.5 (Complex Zeros and the Fundamental Theorem of Algebra)</p>	<ol style="list-style-type: none"> <li>1. <u>Make sense of problems and persevere in solving them.</u></li> <li>3. Reason abstractly and quantitatively.</li> <li>4. <u>Construct viable arguments and critique the reasoning of others.</u></li> <li>5. Model with math.</li> <li>6. <u>Use appropriate tools strategically.</u></li> <li>7. <u>Attend to precision.</u></li> <li>8. Look for and make use of structure.</li> <li>9. Look for and express regularity in repeated reasoning.</li> </ol>
<p>8. Create, interpret and solve problems involving rational functions.</p>	<ol style="list-style-type: none"> <li>1. Determine the unique feature(s) that would one expect to find in the graph of a rational function</li> <li>2. Identify a rational function.</li> <li>3. Distinguish rational functions from polynomial functions.</li> <li>4. Discover the impact the polynomial in the divisor of the rational function has on the domain of the function.</li> <li>5. Interpret asymptotes and relate asymptotes to the domain of the rational function</li> <li>6. Sketch a rational function indicating locations of holes/asymptotes and x and y intercepts.</li> <li>7. Solve a rational equation.</li> <li>8. Interpret and graph models of rational functions as used in real world applications.</li> </ol>	<p>Stewart: Section 3.6 (Rational Functions)</p>	<ol style="list-style-type: none"> <li>1. <u>Make sense of problems and persevere in solving them.</u></li> <li>2. Reason abstractly and quantitatively.</li> <li>3. <u>Construct viable arguments and critique the reasoning of others.</u></li> <li>4. <u>Model with math.</u></li> <li>5. <u>Use appropriate tools strategically.</u></li> <li>6. <u>Attend to precision.</u></li> <li>7. <u>Look for and make use of structure.</u></li> <li>8. Look for and express regularity in repeated reasoning.</li> </ol>



# Precalculus and Function Analysis

Objective	Content Outline Skills and Concepts	Instructional Materials	Standards for Mathematical Practice
9. Represent, evaluate, graph and apply transformations to exponential functions.	<ol style="list-style-type: none"> <li>1. Represent and analyze real world applications of exponential functions.</li> <li>2. Graph exponential functions, and identify domain and range.</li> <li>3. Apply transformations on exponential functions.</li> <li>4. Express an exponential function in standard form.</li> <li>5. Identify and distinguish graphs of exponential functions.</li> <li>6. Recognize the numerical pattern of a logarithmic function.</li> </ol>	<p>Stewart: Section 4.1 (Exponential Functions)</p> <p><i>Calculus: Concepts &amp; Contexts :</i> Section 1.5 (Exponential Functions)</p>	<ol style="list-style-type: none"> <li>1. <u>Make sense of problems and persevere in solving them.</u></li> <li>2. Reason abstractly and quantitatively.</li> <li>3. Construct viable arguments and critique the reasoning of others.</li> <li>4. <u>Model with math.</u></li> <li>5. <u>Use appropriate tools strategically.</u></li> <li>6. Attend to precision.</li> <li>7. Look for and make use of structure.</li> <li>8. Look for and express regularity in repeated reasoning.</li> </ol>

# Precalculus and Function Analysis

Objective	Content Outline Skills and Concepts	Instructional Materials	Standards for Mathematical Practice
<p>10. Recognize, evaluate, graph and apply transformations to logarithmic functions and convert logarithmic functions to exponential functions (and vice versa).</p> <p>11. Manipulate (i.e. expand or combine) and evaluate logarithmic expressions using laws of logarithm.</p> <p>12. Solve exponential and logarithmic equations and apply exponential and logarithmic functions to real-life situations.</p>	<ol style="list-style-type: none"> <li>1. Identify the application of real world models of logarithmic functions and its implementation i.e. invest choices, growth rate, and population analysis.</li> <li>2. Algebraically and graphically construct a logarithm based on interpretation of inverse function to the exponential.</li> <li>3. Recognize the numerical pattern of a logarithmic function</li> <li>4. Explain the relationship between an exponential and logarithmic function and evaluate and convert logs using exponents and exponents using logs.</li> <li>5. Recognize and evaluate a natural log and a common log.</li> <li>6. Recognize and apply the change of base formula.</li> <li>7. Generate equivalent forms for logarithmic and exponential expressions.</li> <li>8. Apply properties of laws of logs to evaluate logarithms.</li> <li>9. Graph logarithmic functions and given transformations.</li> <li>10. Transform logarithmic functions.</li> <li>11. Find the domain of a logarithmic function.</li> <li>12. Implement the use of logs and exponents to solve equations and real world models and applications</li> </ol>	<p>Stewart: Section 4.2(Logarithmic Functions) Section 4.3(Laws of Logarithms) Section 4.4 (Exponential and Logarithmic Equations)</p> <p><i>Calculus: Concepts &amp; Contexts :</i> Section 1.6 (Inverse Functions and Logarithms)</p>	<ol style="list-style-type: none"> <li>1. <u>Make sense of problems and persevere in solving them.</u></li> <li>2. Reason abstractly and quantitatively.</li> <li>3. <u>Construct viable arguments and critique the reasoning of others.</u></li> <li>4. <u>Model with math.</u></li> <li>5. <u>Use appropriate tools strategically.</u></li> <li>6. <u>Attend to precision.</u></li> <li>7. <u>Look for and make use of structure.</u></li> <li>8. Look for and express regularity in repeated reasoning.</li> </ol>

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Objective	Content Outline Skills and Concepts	Instructional Materials	Standards for Mathematical Practice
13. Evaluate, analyze and relate the concept of a limit and estimate the value of a given limit numerically, graphically and algebraically.	<ol style="list-style-type: none"> <li>Investigate, identify and evaluate the limit of a function given its graph and its equation.</li> <li>Investigate, identify values at which the limit of a function does not exist.</li> <li>Determine limits in a piecewise function.</li> <li>Apply piecewise functions to interpret the meaning and importance of one-sided limits.</li> <li>Sketch a function given information about its limits</li> <li>Use a graphing calculator to evaluate limits of a function</li> <li>Determine limits using direct substitution and factoring.</li> <li>Evaluate limits using limit laws</li> <li>Evaluate limits by algebraic simplification, by rationalization of terms and by direct substitution.</li> <li>Determine the appropriate approach to evaluating a limit based off of a given function and identify when we can use direct substitution and when we cannot.</li> </ol>	<p>Stewart:            Section 12.1 (Finding Limits Numerically and Algebraically)            Section 12.2 (Finding Limits Algebraically)</p> <p><i>Calculus: Concepts &amp; Contexts :</i>            Section 2.2 (The Limit of Function)            Section 2.3 (Calculating Limits Using Limit Laws)</p>	<ol style="list-style-type: none"> <li>Make sense of problems and persevere in solving them.</li> <li><u>Reason abstractly and quantitatively.</u></li> <li><u>Construct viable arguments and critique the reasoning of others.</u></li> <li><u>Model with math.</u></li> <li><u>Use appropriate tools strategically.</u></li> <li><u>Attend to precision.</u></li> <li><u>Look for and make use of structure.</u></li> <li><u>Look for and express regularity in repeated reasoning.</u></li> </ol>

# Precalculus and Function Analysis

Timeframe	Sequencing and Pacing
Quarter 1	<p>Unit 1: Fundamentals</p> <ul style="list-style-type: none"> <li>1.a Algebraic Notation and Manipulation (w/ focus on factoring)</li> <li>1.b Rational Expressions</li> <li>1.c Linear and Quadratic Equations</li> </ul> <p>Unit 2: Functions</p> <ul style="list-style-type: none"> <li>2.a Recognizing and evaluating functions</li> <li>2.b Graphs of Functions</li> <li>2.c Average Rate of Change</li> <li>2d. Domain of Given Functions</li> </ul> <p>Unit 3: Polynomial Functions</p> <ul style="list-style-type: none"> <li>3.a Attributes of Polynomial Functions and their Graphs</li> <li>3.b Long and Synthetic Polynomial Division</li> <li>3.c Finding Zeros of Polynomials</li> <li>3.d Complex Number Operations and Solutions</li> <li>3.e Fundamental Theorem of Algebra</li> <li>3.f Attributes of Rational Functions and their Graphs</li> </ul>
Quarter 2	<p>Unit 4: Exponential and Log Functions</p> <ul style="list-style-type: none"> <li>4.a Exponential Functions</li> <li>4.b Logarithmic Functions</li> <li>4.c Laws of Logarithms</li> <li>4.d Solving Exponential and Logarithmic Equations</li> <li>4.e Modeling with Exponential and Logarithmic Functions</li> </ul> <p>Unit 5: Limits: A Preview of Calculus</p> <ul style="list-style-type: none"> <li>5.a Finding Limits Numerically and Graphically</li> <li>5.b Finding Limits Algebraically</li> <li>5.c Instantaneous Slope and Tangent Lines</li> </ul>